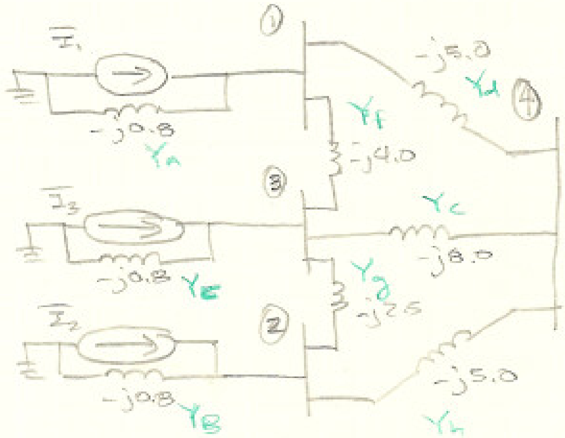
continued example from last day.



where

$$Y_a = \frac{1}{j1.15 + j0.1} = -j0.8 \text{ PU}$$

etc

the net injected curren into node 1

$$I_1 = V_1 Y_A + (V_1 - V_3) Y_f + (V_1 - V_4) Y_d$$

similarly for nodes 2 & 3
for node 4 we have

$$0 = (V_4 - V_1) Y_d + (V_4 - V_2) Y_h + (V_4 - V_3) Y_e$$

for all nodes in matrix form.

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{Y}_a + \bar{Y}_f + \bar{Y}_d & 0 & -\bar{Y}_f & -\bar{Y}_d \\ 0 & \bar{Y}_h + \bar{Y}_g + \bar{Y}_b & -\bar{Y}_g & -\bar{Y}_h \\ -\bar{Y}_f & -\bar{Y}_g & \bar{Y}_f + \bar{Y}_c + \bar{Y}_g + \bar{Y}_e & -\bar{Y}_e \\ -\bar{Y}_d & -\bar{Y}_h & -\bar{Y}_e & \bar{Y}_d + \bar{Y}_e + \bar{Y}_h \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$\uparrow \bar{I}_4 = 0$

So in general, we may write for n nodes.

$$\boxed{\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & & & \vdots \\ \vdots & & & \vdots \\ Y_{n1} & \cdots & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}}$$

In summation form $\overline{I}_k = \sum_{i=1}^{n} \overline{Y}_{ki} \overline{V}_i$ for $k = 1, 2, \ldots, n$

In our previous example, let:

$E_1 = 1.5 \angle 0^\circ$ PU Volts

$E_2 = 1.5 \angle -36.87^\circ$ PU Volts

$E_3 = 1.5 \angle 0^\circ$ PU Volts.

we write our set of currents as

$$\overline{I}_1 = \overline{I}_2 = \frac{1.5}{1.25} \angle 0^\circ = 1.2 \angle -90^\circ = 0 - j1.2 \text{ PU}$$

$$\overline{I}_3 = \frac{1.5}{j1.25} \angle -36.87^\circ = 1.2 \angle -126.87^\circ \text{ PU.}$$

$$= -0.72 - j0.96 \text{ PU}$$

self admitances

$\overline{Y}_{11} = -j5.0 - j0.4 - j0.8 = -j9.8$

$\overline{Y}_{22} = -j5.0 - j2.5 - j0.8 = -j8.3$

$\overline{Y}_{33} = -j4.0 - j2.5 - j8.0 - j0.8 = -j15.3$

$\overline{Y}_{44} = -j5.0 - j5.0 - j8.0 = -j18.0$

mutual off diagonal admittances in PU are.

$$Y_{12} = Y_{21} = 0$$
$$Y_{23} = Y_{32} = j2.5$$
$$Y_{13} = Y_{31} = j4.0$$
$$Y_{24} = Y_{42} = j5.0$$
$$Y_{14} = Y_{41} = j5.0$$
$$Y_{34} = Y_{43} = j8.0$$

then the vector matrix form,

$$\begin{bmatrix} 0-j1.2 \\ -0.72-j96 \\ 0-j1.2 \\ 0 \end{bmatrix} = \begin{bmatrix} -9.8j & 0 & j4 & j5 \\ 0 & -j8.3 & j2.5 & j5 \\ j4 & j2.5 & -j15.3 & j8 \\ j5 & j5 & j8 & -j18 \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \end{bmatrix}$$

to solve for the voltage vector, we need to invert the matrix $\underline{Y}$.

EX:

$$\bar{V} = \underline{Y}^{-1} \bar{I}$$

The above process involves matrix Inversion.

MATRIX PARTIONING

a very useful technique of matrix manipulation is known as matrix partioning.

consider the following 3×3 matrix equation

$$\begin{bmatrix} Ⓜ C_1 \\ C_2 \\ Ⓝ C_3 \end{bmatrix} = \begin{bmatrix} a_{11} & Ⓓ a_{12} & a_{13} Ⓔ \\ a_{21} & a_{22} & a_{23} \\ a_{31} & Ⓕ a_{32} & a_{33} Ⓖ \end{bmatrix} \begin{bmatrix} b_1 Ⓗ \\ b_2 \\ b_3 Ⓘ \end{bmatrix}$$

$$\underline{C} = \underline{A}\,\underline{B}$$

our partitioned form is:

$$\boxed{\underline{C} = \begin{bmatrix} \underline{M} \\ \underline{N} \end{bmatrix} = \begin{bmatrix} \underline{D} & \underline{E} \\ \underline{F} & \underline{G} \end{bmatrix} \begin{bmatrix} \underline{H} \\ \underline{J} \end{bmatrix}}$$

or we can write,

$$\boxed{\underline{M} = \underline{D}\,\underline{H} + \underline{E}\,\underline{J}}$$

$$\boxed{\underline{N} = \underline{F}\,\underline{H} + \underline{G}\,\underline{J}}$$

NODE ELIMINATION BY MATRIX PARTITIONING

often in solving power system problems of the form

$$\underline{I}_{bus} = \underline{Y}_{bus}\,\underline{V}_{bus}$$

we don't need to solve for these nodes where the injected currents into them are zero.

so we may then use matrix partitioning to solve for the rest of the nodes.

consider the following partitioned matrix eqn.

$$\boxed{\begin{bmatrix} \underline{I}_A \\ \underline{I}_x \end{bmatrix} = \begin{bmatrix} \underline{K} & \underline{L} \\ \underline{L}^T & \underline{M} \end{bmatrix} \begin{bmatrix} \underline{V}_A \\ \underline{V}_x \end{bmatrix}}$$

let us assume that $\underline{I}_x$ is the subvector containing ZERO injected currents and $\underline{V}_x$ is there corresponding bus voltage subvector

$$\underline{I}_A = \underline{K}\,\underline{V}_A + \underline{L}\,\underline{V}_x$$

$$\underline{I}_x = \underline{L}^T\underline{V}_A + \underline{M}\,\underline{V}_x$$